# A MODEL OF BUBBLE CAVITATION IN A REAL LIQUID

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Present-day notions about the mechanism of the evolution of a cavitation cluster, i.e., a swarm of minute vapor–gas bubbles, in a sound field (e.g., in the focal region of a "concentrator" or acoustic velocity transformer) are based on the instability of the shape of the bubbles during their explosionlike expansion and rapid collapse (implosion). It is assumed that instability induces bubble disintegration and, hence, an avalanche-type multiplication of cavitation centers [1]. It has been postulated, on the basis of an analysis of high-speed motion pictures of the process [1], in particular, that the number of bubbles in the visible cavitation zone can be many orders of magnitude greater than the number of initial cavitation nuclei as determined, e.g., according to Gavrilov's procedure [2]. Experiments indicate that this effect is dynamic: During the first few periods after application of the sound field, the number of bubbles is consistent with the number of nuclei expected according to the state of the liquid, but then it increases and arrives at a steady state, which is governed by the characteristics of the field and the liquid [1].

However, it is difficult within the framework of this approach to explain the occurrence of a uniform spatial distribution of the fragments of a bubble when it is disintegrated as a result of instability [1]. On the other hand, in underwater-explosion experiments, e.g., an instability is also recorded in the shape of the cavity containing the detonation products, which resembles a bubble swarm at the instant of its compression. But this formation does not subsequently disintegrate into separate elements and is certainly not uniformly distributed in the near field [3]. Another phenomenon can be cited, viz.: A zone of intense bubble cavitation emerges near a free surface when a shock wave is reflected from it, despite the fact that the only stimulus is a single rarefaction pulse [4] and "ultrasonic pumping" of the zone of the nuclei (by an instability mechanism) is absent. The abrupt visible variation in the number density of bubbles in a zone near the surface of an underwater acoustic transducer as it approaches a rigid wall can be classified as a phenomenon of the same order [5, 6]. The maximum radii of the visible cavities in this case are much smaller than for a distant transducer [5].

Obviously, the state of the liquid in the cited experiments is recorded at a time when the cavitation nuclei reach a visible size corresponding to resolution with a definite degree of accuracy. Ultrasonic cavitation experiments have shown that the cavitation cluster pulsates, vanishing periodically from the field of view [6]; the bubbles collapse to a size below the resolving power of the instrumentation. Thus, the inception and dynamics of the cavitation zone are viewed as being associated with the "lifetime" of a bubble of visible radius.

In this article we investigate the conditions and time for the bubbles to reach a visible size in the case of a broad theoretically possible spectrum with respect to their initial radius.

A theoretical analysis of the cavitation–initiation process under the action of a negative pulse of constant amplitude is carried out within the framework of the definitions of [7, 8] in the example of single-bubble dynamics. Cavitation is considered to be fully developed when the bubbles attain a radius of $\sim 10^{-2}$ cm, which is taken as the visibility threshold. It is shown that the presence of gas in the bubbles has a definite influence on the threshold values of the pressure difference capable of initiating their unbounded growth.

## Statement and Qualitative Analysis of the Problem

The dynamics of a spherical bubble in an unbounded ideal incompressible liquid is known to be described by the equation

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$$a\ddot{a}+\frac{3}{2}\dot{a}^2=\frac{1}{\rho}\left[\left(p_0+\frac{2\sigma}{a_0}\right)\left(\frac{a_0}{a}\right)^{3\gamma}-\frac{2\sigma}{a}-p_\infty\right], \tag{1}$$

where $\rho$ is the density of the liquid, $a$ is the bubble radius, the index 0 indicates the initial values of the radius and the hydrostatic press, $\sigma$ is the coefficient of surface tension, $\gamma$ is the polytropy index of the gas, and $p_\infty$ is the pressure at infinity. It is assumed that at time $t = 0$ the quantities $a(0) = a_0$, $\dot{a}(0) = 0$, and $p_\infty$ drops instantaneously by the amount $\Delta p$, i.e., $p_\infty = p_0 - \Delta p$. It is required to determine the spectrum of initial radii of bubbles that can attain visible size for a given $p_\infty$, along with the time for them to attain that size.

It is convenient to introduce dimensionless variables and parameters according to the expressions $R = a/a_V$, $R_0 = a_0/a_V$, $V = R^3$, $V_0 = R_0{}^3$, $t' = tc_0/a_V$, $p = p_\infty/p_0$, $We = 2\sigma/p_0 a_V$, $\eta = p_0/\rho_0 c_0{}^2$, where $a_V$ is the just-visible bubble radius and $c_0$ is the unperturbed sound velocity in the liquid. Equation (1) is rewritten in the form

$$R\ddot{R}+\frac{3}{2}\dot{R}^2=\eta\left[(1+\mathrm{We}/R_0)(R_0/R)^{3\gamma}-p-\mathrm{We}/R\right]. \tag{2}$$

The first integral of Eq. (2) with respect to the bubble volume V is already obtained:

$$V^{-1/3}\dot{V}^2=6\eta F(V;\,V_0,\,p). \tag{3}$$

Here, if $\gamma \neq 1$,

$$F=\frac{1+\mathrm{We}\,V_0^{-1/3}}{\gamma-1}V_0^\gamma\left(V_0^{1-\gamma}-V^{1-\gamma}\right)-p\,(V-V_0)-\frac{3}{2}\mathrm{We}\left(V^{2/3}-V_0^{2/3}\right),$$

and if $\gamma = 1$,

$$F=\left(1+\mathrm{We}\,V_0^{-1/3}\right)V_0\ln(V/V_0)-p\,(V-V_0)-\frac{3}{2}\mathrm{We}\left(V^{2/3}-V_0^{2/3}\right).$$

The right-hand side of Eq. (3) describes a family of curves that depends on the parameters $V_0$ and p. Clearly, a solution can exist for it only for intervals of the curves in which $F \geq 0$. For the quantitative analysis of the possible solutions it is useful to write the derivative

$$F_V=\left(1+\mathrm{We}\,V_0^{-1/3}\right)(V_0/V)^\gamma-\mathrm{We}\,V^{-1/3}-p.$$

The functions F and $F_V$ have the properties

$$F(0;\,V_0,\,p)=-\infty,\ F(V_0;\,V_0,\,p)=0,\ F_V(0;\,V_0,\,p)=\infty,$$
$$F_V(V_0;\,V_0,\,p)=1-p,\ F_V(\infty;\,V_0,\,p)=-p.$$

We partition the interval of values of p into two subintervals: $0 \leq p < 1$ and $p < 0$. For $p \geq 0$ (the form of the function F is represented by curve 1 in Fig. 1a), the bubble volume oscillates between the values $V_0$ and $V_2$. If $V_2 = 1$, we deduce the following expression from the condition $F(1;\,V_0,\,p) = 0$ (curve DE in Fig. 1b):

$$p=\frac{1+\mathrm{We}\,V_0^{-1/3}}{\gamma-1}V_0^\gamma\frac{V_0^{1-\gamma}-1}{1-V_0}-\frac{3}{2}\mathrm{We}\frac{1-V_0^{2/3}}{1-V_0}=f(\mathrm{We},\,V_0),$$

which in the pulsation regime determines the threshold at which the bubbles reach visible size, i.e., for $p \leq f(\mathrm{We}, V_0)$ (domain $\Omega_1$ in Fig. 1b) all the bubbles reach visible size, and for $p > f(\mathrm{We}, V_0)$ (domain $\Omega_2$) they do not. Let $p < 0$. The existence of threshold pressures, at which unbounded bubble growth takes place and the so-called unbounded-growth curve is obtained, has been proved [8]. The indicated curve is specified in the parametric form

$$F(V_*;\,V_0,\,p)=0,\quad F_V(V_*;\,V_0,\,p)=0 \tag{4}$$

with the parameter $V_*$ (Fig. 1c) and is represented schematically by curve BAC in Fig. 1b. This curve determines the threshold above which the bubble grows without bound. The value $V = V_*$ corresponds to the maximum radius, toward which the bubble tends asymptotically in infinite time. In the $V_0p$ plane, curve (4) separates the domain $\Omega_3$ of periodic bubble pulsa-

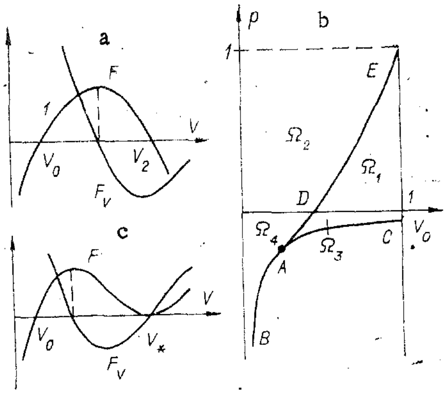


Fig. 1

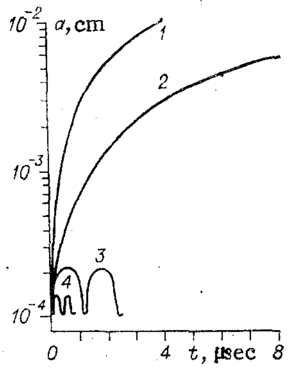


Fig. 2

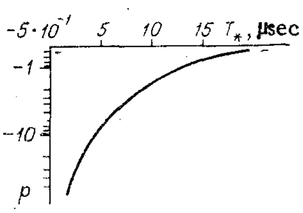


Fig. 3

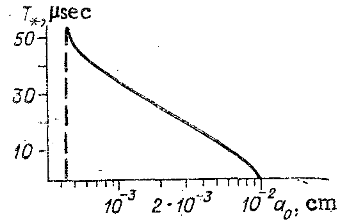


Fig. 4

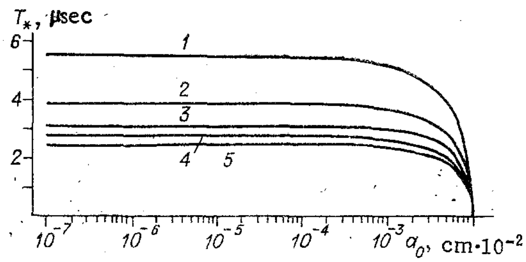


Fig. 5

tions from the domain $\Omega_4$ in which the bubble grows without bound and, hence, necessarily reaches visible size from the point of view of the given model. The bubbles pulsate in the domain $\Omega_3$ in this case, reaching visible size. It can be shown that $F(1;\ V_0,\ p) = 0$ is tangent to the unbounded-growth curve BAC at point A, where $V_* = 1$. Accordingly, BAE represents the boundary of the domain in which the bubbles reach visible size for all possible values of p.

## Results of Calculations

The calculations of the microbubble dynamics were carried out according to Eq. (2). The time for a bubble to grow to visible size is given by the integral

$$t = \int_{V_0}^{1} \frac{V^{-1/6}dV}{\sqrt{6\eta F}}.$$

In the example of a nucleus with initial radius $a_0 = 10^{-4}$ cm, Fig. 2 shows typical features of its behavior in dimensioned form for various values of the negative amplitudes of the tensile stresses relative to $p_0 = 10^5$ Pa: 1) p = −10; 2) −1; 3) −0.5; 4) −0.1. As the tensile stresses are increased, we observe a transition from nonvisible pulsations (curves 3 and 4) to unbounded expansion with attainment of the visible radius $a_V = 10^{-2}$ cm (curves 1 and 2). Figure 3 shows the time $T_*$ for the bubble to reach visible size as a function of the tensile-stress amplitudes ($a_0 = 10^{-4}$ cm). These data evince a rather steep gradient of the function in a narrow zone of values of p close to the threshold of asymptotic bubble

TABLE 1

| $a_0$, cm | $R_V$ | $\tau_*$ |
|---|---|---|
| $3.16\cdot10^{-3}$ | 1.58—3.16 | 3.25—31 |
| $10^{-3}$ | 5—10 | 5—18 |
| $3.16\cdot10^{-4}$ | 15.8—31.6 | 9—13 |
| $10^{-4}$ | 50—61.6 * | 10.8—12.43 |

growth. For example, $T_* = 18.8$ μsec at $p = -0.55$, and at $p = -0.5$ (curve 3 in Fig. 2) the nucleus pulsates with $a_{max} \approx 2a_0$, while at $p \approx -0.516$ its radius tends asymptotically to the value $a \approx 3a_0$, i.e., it essentially does not reach visible size.

The calculation of the time $T_*$ for a cavitation nucleus to reach the visible radius $a_V$ has made it possible to discern two fundamental features: For small negative tensile-stress amplitudes, $T_*$ depends significantly on $a_0$; for large negative amplitudes, nuclei with practically the entire range of initial radii attain the visible radius simultaneously.

Indeed, for $p = -0.1$ (Fig. 4), nuclei with an initial radius $a_0 = 5\cdot10^{-3}$ cm attain visible size in 12.5 μsec, and those with $a_0 = 5\cdot10^{-4}$ cm attain it in 47.5 μsec. The dashed line represents the boundary at which the initial radius of a nucleus can grow to the visible radius for a given value of p. We see that this line is the asymptote of the function $T_*(a_0)$: $T_* \to \infty$ for $a_0 \approx 4.3\cdot10^{-4}$ cm.

Figure 5 shows curves of the function $T_*(a_0)$ for relatively strong unloading: 1) $p = -5$; 2) $-10$; 3) $-15$; 4) $-20$; 5) $-25$. Comparing the data of Figs. 2, 3, and 5, we infer that unloading with a constant amplitude $p = -5$ representing the "upper bound" can be regarded as the threshold at which practically the entire spectrum of initial cavitation-bubble radii attain the visible radius $a_V = 10^{-2}$ cm simultaneously. In a real situation, the cavitation zone transforms the applied stress field.

Both of the indicated features have a direct bearing on the visible chain-reaction effect of the "multiplication" of cavitation centers, the mechanism of which is determined not so much by instability of the shape of the pulsating cavitation bubbles and their disintegration as by the possibility and time lag of the process of small bubbles reaching visible size from the spectral composition of the cavitation nuclei. Of course, the definition of the "visible" bubble radius is conditional, because it must be a quantity that can be reliably resolved within the scope of the particular experimental procedure in every specific situation.

These considerations and conclusions apply to ultrasonic cavitation to a certain extent. As an example, we consider data from an investigation [9] of single-bubble dynamics in an ultrasonic field specified in the form $p_\infty = p_0 + Ap_0 \cos \omega t$, where $A > 0$ and the process begins in the compression phase (quarter period). For $A = 1.5$ (corresponding to $p = -0.5$ at the maximum of the negative phase) and $\omega \approx 116$ kHz, the visibility-attainment time of the nuclei varies in the interval $T_* \approx 20$-$42$ μsec for a variation of their radii from $8.6\cdot10^{-3}$ to $8.6\cdot10^{-4}$ cm. Visibility is attained practically in the first unloading phase (wave period of 54 μsec). With an increase in A, even with a simultaneous increase in the frequency ω of the field, a tendency is observed for a fairly broad spectrum of cavitation nuclei to reach visible size simultaneously. For example, in the case $A = 5$ and $\omega = 450$ kHz, the visibility-attainment time varies only in the interval 7.4-12.3 μsec for a 30-fold variation in the value of $a_0$ ($a_0 = 7\cdot10^{-3}$ to $2.2\cdot10^{-4}$ cm).

Because of a certain indeterminacy in the values of $a_V$, it is convenient to analyze the distinctive features of the process of bubbles in a given size range attaining visibility. An interesting effect is observed in this case for $A = 5$ and $\omega = 10^6$ Hz for the interval $a_V = 5\cdot10^{-3}$ to $10^{-2}$ cm.

Table 1 shows the intervals of visibility-attainment times $\tau_* = \omega T_*$ for various intervals of relative just-visible bubble radii $R_V = a_V/a_0$. The asterisk indicates the case $R = R_{max}$. For large initial radii of the cavitation nuclei, $\tau_*$ increases abruptly (by an order of magnitude) as the values of $a_V$ vary in the visible interval from 50 to 100 μm. With a decrease in $a_0$, the interval for $\tau_*$ becomes narrower with an increase in the lower bound and a decrease in the upper bound. It can be expected on this basis that the time to attain visible size will stabilize in approximately two periods of bubbles having a radius of 1 μm or smaller. It is important to note the explicit dependence of the visibility-attainment time on the initial radius: For $R_0$ = 31.6 μm the bubbles reach the lower bound of visible radii in approximately half a period, and for $R_0$ = 1 μm they do so at the end of the second period.

The foregoing analysis shows that three characteristic types of bubble dynamics occur in a field of constant tensile stresses: oscillations; monotonic growth to an asymptotic value in infinite time; unbounded growth. As a result of the calculations, we have plotted the visibility-attainment curve, which determines the threshold pressures initiating the growth of bubbles to visible size. It is found that the time for a bubble to attain visible size depends significantly on $\alpha_0$ for small applied-unloading amplitudes. The interval of values of $\alpha_0$ admitting such an attainment is finite. For large amplitudes, bubbles having the entire spectrum of initial radii are observed to reach visible size practically simultaneously. Bubbles in a high-frequency ultrasonic field are characterized by similar effects.

Thus, if inhomogeneities at which a cavitation cluster can develop exist in a real liquid with a number density of the order of $10^5$ to $10^6$ $cm^{-3}$, the evolution of the cluster with time into a visible structure is determined by the nature and parameters of the rarefaction phases of the wave field and is manifested either in "instantaneous" arrival at the maximum bubble density (as in the case of reflection of a strong shock wave from a free surface) or in the gradual saturation of the cluster with bubbles as a result of the successive attainment of the visibility zone by smaller and smaller nuclei.

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